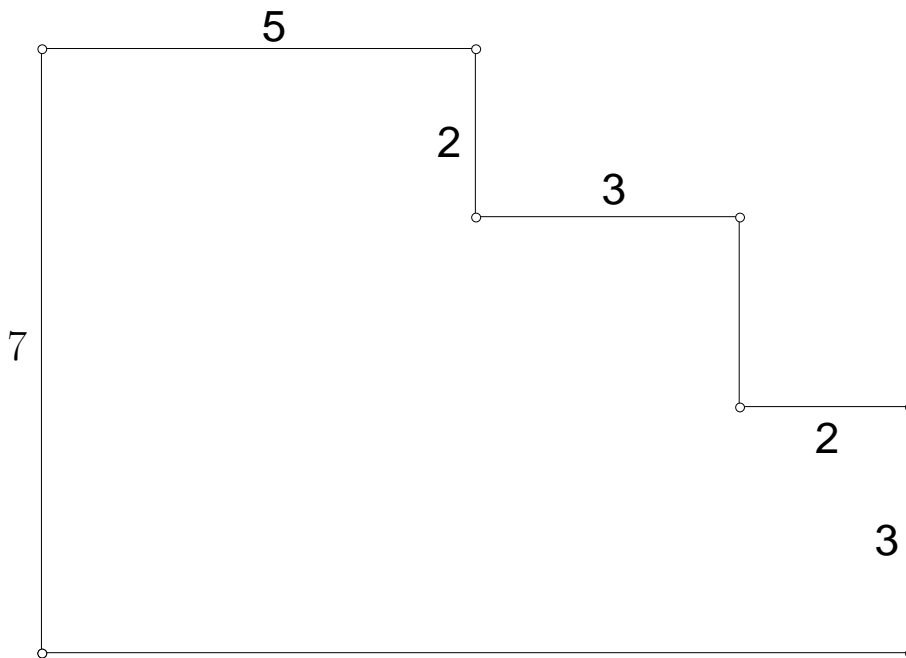


# Merten Hasse Math Competition Geometry Test 2004

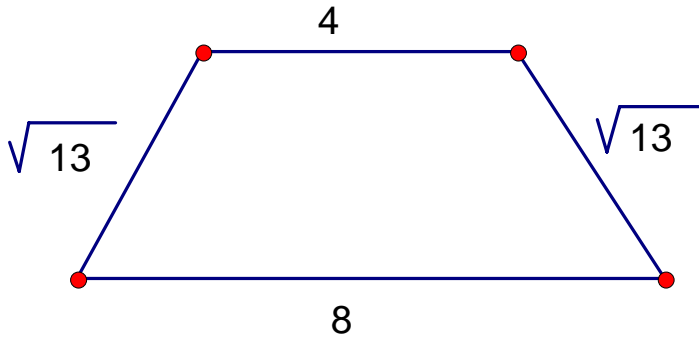
1. Given that the adjacent sides are perpendicular to one another, find the enclosed area in the figure to the right.

- a) 48    b) 50  
c) 56    d) none of the previous



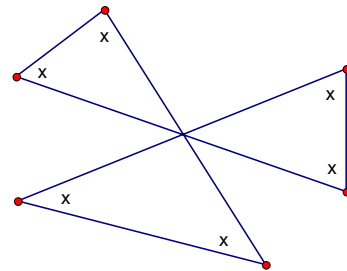
2. Find the volume in cubic units of the pyramid that has the trapezoid base shown to the right and a height of 6 units.

- a)  $24 + 4\sqrt{13}$       b) 36  
 c) 48                      d) none of the previous



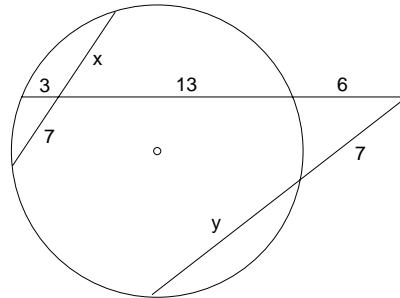
3. Find the sum of the marked angles as shown in the figure to the right. Note there are three lines intersecting at a point.

- a)  $360^{\circ}$       b)  $300^{\circ}$   
 c)  $180^{\circ}$       d) none of the previous



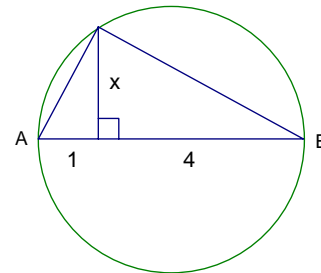
4. Find the value of  $y$  in the figure to the right.

- a)  $y = \frac{22}{7}$       b)  $y = \frac{83}{7}$   
 c)  $y = \frac{96}{7}$       d) none of the previous



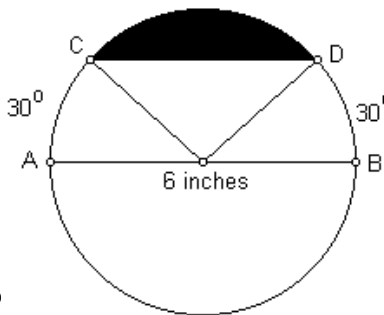
5. Find the value of  $x$  in the figure to the right where  $\overline{AB}$  is a diameter of the given circle.

- a)  $x = \sqrt{3}$       b)  $x = 2$   
 c)  $x = \sqrt{5}$       d) none of the previous



6. Find the sum of the interior angles of a regular heptagon.
- a)  $1260^\circ$       b)  $1080^\circ$
- c)  $900^\circ$       d) none of the previous
7. Find the area of a regular hexagon if the perpendicular distance from one of its sides to the center is 4 feet.
- a)  $32 \text{ feet}^2$       b)  $32\sqrt{2} \text{ feet}^2$
- c)  $32\sqrt{3} \text{ feet}^2$       d) none of the previous

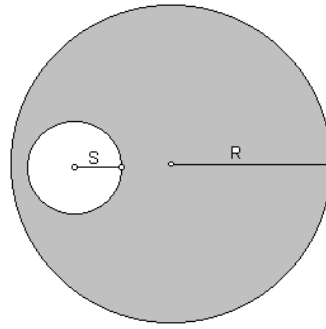
8. Find the area of the shaded region for the circle at the right where the diameter is 6 inches,  $\widehat{AC} = 30^\circ$ , and  $\widehat{BD} = 30^\circ$ .



- a)  $(2\pi - 9\sqrt{3}) \text{ in}^2$       b)  $(3\pi - \frac{9\sqrt{3}}{2}) \text{ in}^2$
- c)  $(3\pi - 9\sqrt{3}) \text{ in}^2$       d) none of the previous
9. Find the total surface area for a cone with a base radius of 5 in and an altitude of 12 in.
- a)  $90\pi \text{ in}^2$       b)  $85\pi \text{ in}^2$
- c)  $75\pi \text{ in}^2$       d) none of the previous
10. Find the area of a rhombus whose sides are of length  $5m$  and has one diagonal of length  $6m$ .
- a)  $24m^2$       b)  $25m^2$
- c)  $36m^2$       d) none of the previous
11. Find an equation of the circle whose diameter endpoints have the coordinates  $(-2, 5)$  and  $(16, 21)$ .
- a)  $(x - 7)^2 + (y - 13)^2 = 145$       b)  $(x + 7)^2 + (y + 13)^2 = 290$
- c)  $(x - 7)^2 + (y - 13)^2 = 290$       d) none of the previous

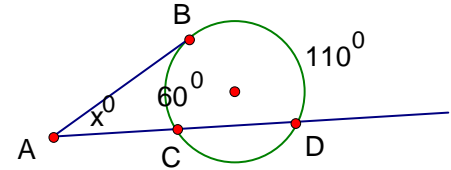
12. What is the area of the shaded region in the figure to the right?

- a)  $\pi(R - S)^2$       b)  $2\pi R - 2\pi S$   
 c)  $\pi R^2 - \pi S^2$       d) none of the previous



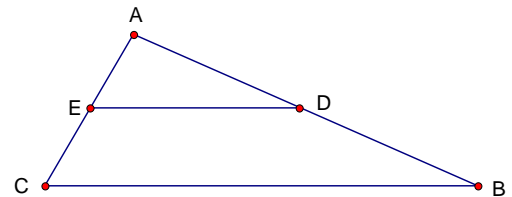
13. Find the value of  $\angle BAC$  for the figure to the right where  $\overline{AB}$  is tangent to the given circle,  $\widehat{BC} = 60^\circ$ , and  $\widehat{BD} = 110^\circ$ .

- a)  $x = 85$       b)  $x = 30$   
 c)  $x = 25$       d) none of the previous



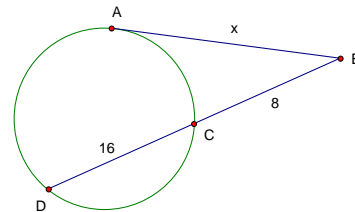
14. Given  $\triangle ABC$  and  $D$  is the midpoint of  $\overline{AB}$  and  $E$  is the midpoint of  $\overline{AC}$ . Find  $DE$  if  $AC = 10$ ,  $AB = 21$ , and  $BC = 23$ .

- a) 11.5      b) 11  
 c) 10.5 cm      d) none of the previous



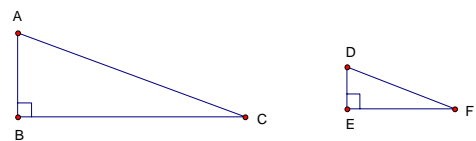
15. Given  $\overline{AB}$  is tangent to the circle,  $BC = 8$ , and  $CD = 16$ , what is the value of  $AB$  to the nearest tenth?

- a) 16.0      b) 13.9  
 c) 8.1      d) none of the previous

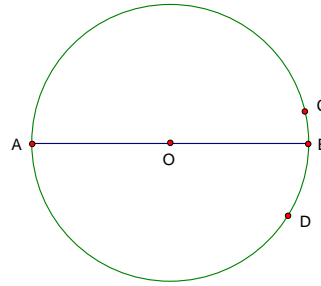


16. Given that  $\triangle ABC \sim \triangle DEF$  with ratio 2,  $\angle A = \angle D = 60^\circ$ ,  $\angle B = \angle E = 90^\circ$ , and  $DE = 2$ , find the length of  $\overline{BC}$ .

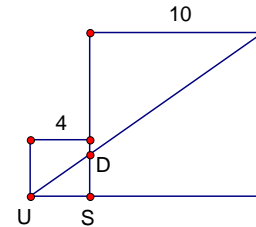
- a) 2      b) 4  
 c) 8      d) none of the previous



17. Given the circle with center  $O$ , diameter  $\overline{AB}$ , and with points on the circle such that  $\widehat{DBC} = 40^\circ$  and  $\widehat{AC} = 170^\circ$ , what is the measure of  $\widehat{AD}$ .



- a)  $120^\circ$     b)  $140^\circ$   
 c)  $160^\circ$     d) none of the previous
18. Given the two squares with sides of length 4 and 10, calculate the area of  $\triangle USD$  as shown to the right.



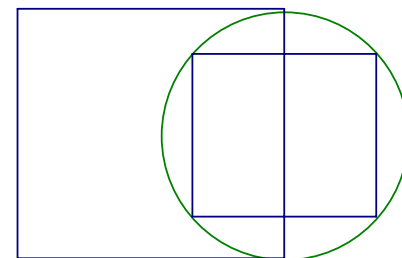
- a)  $50/7$     b)  $45/7$   
 c)  $40/7$     d) none of the previous

19. If I increase the height of a triangle by 25% and decrease its base by 25%, what will happen to the area of the triangle?
- a) Increases by a factor more than  $5/4$ .    b) Decreases by a factor between  $7/8$  and 1.  
 c) Stays the same.    d) none of the previous

20. I measure the shadow of a tree and my shadow at the same time and find that my shadow is 8.5 ft. and the tree's shadow is 102 feet. Given that I am 6 ft 4 inches tall, find how tall the tree is.

- a) 75 ft    b) 888 inches  
 c) 76 ft    d) none of the previous

21. Find the ratio of a side of the large square to a side of the small square in the figure to the right.



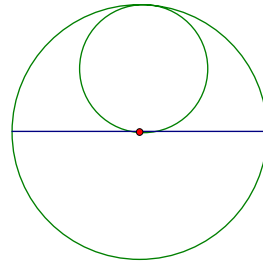
- a)  $\sqrt{2}/2$     b) 2  
 c)  $\sqrt{2}$     d) none of the previous

22. A chord of a circle is the hypotenuse of an isosceles right triangle whose legs are radii of the circle. The radius of the circle is  $4\sqrt{2}$ . Find the length of the minor arc subtended by the chord.

- a)  $4\pi\sqrt{2}$     b)  $7\pi\sqrt{2}$   
 c) 12            d) none of the previous

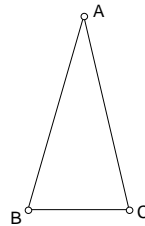
23. A circle is inscribed in a semicircle. The diameter of the smaller circle and the radius of the semicircle are 12 units. Find the area of the region of the semicircle that is outside the smaller circle.

- a) 36    b) 24  
 c) 12    d) none of the previous



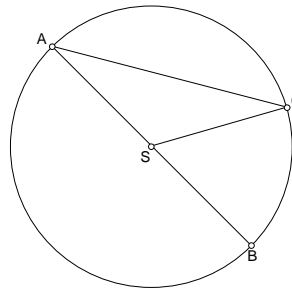
24. Given isosceles triangle  $\triangle ABC$  where the measure of  $\angle A$  is half of the measure of  $\angle B$ , find the measure of  $\angle A$ .

- a)  $36^\circ$     b)  $72^\circ$   
 c)  $144^\circ$     d) none of the previous



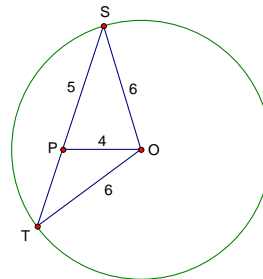
25. Given circle  $S$  with diameter  $\overline{AB}$  and  $\angle A = 30^\circ$ , find the measure of  $\angle CSB$ .

- a)  $30^\circ$     b)  $45^\circ$   
 c)  $60^\circ$     d) none of the previous



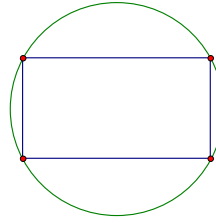
26. Given a circle of radius 6 with center  $O$ ,  $OP = 4$ , and  $PS = 5$ . Find the length of  $\overline{PT}$ .

- a) 2    b) 4  
 c) 6    d) none of the previous



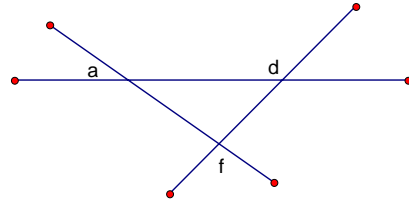
27. Find the area of the circle if the dimensions of the inscribed rectangle are  $8 \times 14$ .

- a)  $50\pi$     b)  $65\pi$   
 c)  $68\pi$     d) none of the previous



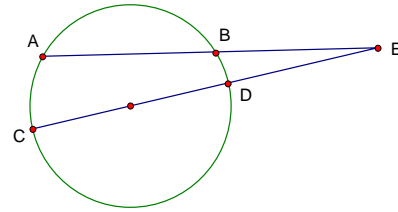
28. In the figure to the right,  $m\angle a = 45^\circ$  and  $m\angle d = 145^\circ$ . Find  $m\angle f$ .

- a)  $90^\circ$     b)  $100^\circ$   
 c)  $110^\circ$     d) none of the previous



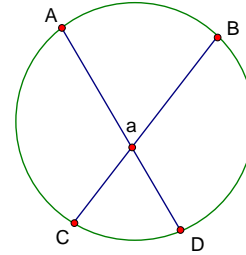
29. Given that  $\overline{CD}$  is a diameter of the given circle,  $\widehat{AB} = 74^\circ$ , and  $\widehat{CA} = 70^\circ$ , find  $m\angle E$ .

- a)  $15^\circ$     b)  $17^\circ$   
 c)  $19^\circ$     d) none of the previous



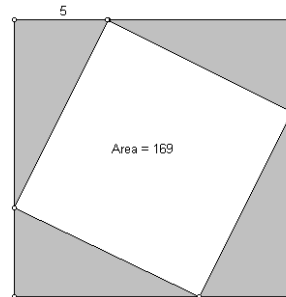
30. Given that the chords  $\overline{AD}$  and  $\overline{BC}$  intersect as shown,  $\widehat{CD} = 20^\circ$ ,  $\angle a = 55^\circ$ , and  $\widehat{BD} = \widehat{AC}$ , find the measure of  $\widehat{BD}$ .

- a)  $90^\circ$     b)  $125^\circ$   
 c)  $180^\circ$     d) none of the previous



31. Consider the 2 squares constructed so that they form four congruent triangles as shown to the right. If the area of the small square is 169 units and the leg of a triangle is given as 5, find the area of the shaded region.

- a) 120    b) 180  
 c) 240    d) none of the previous

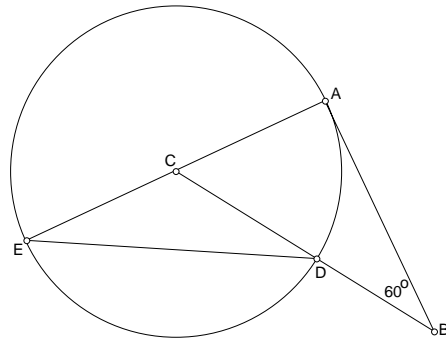


32. What is the area of a square that has a circumscribed circle with an area of  $9\pi$ ?

- a) 36    b) 27  
 c) 18    d) none of the previous

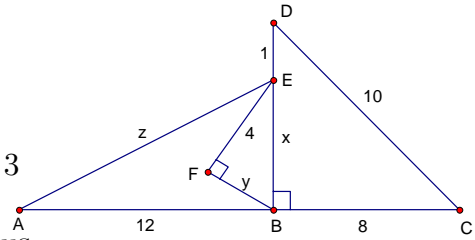
33. Given a circle with center  $C$  and tangent segment  $\overline{AB}$ . If  $\angle B = 60^\circ$ , find the measure of  $\angle AED$ .

- a)  $15^\circ$     b)  $30^\circ$   
 c)  $60^\circ$     d) none of the previous



34. Given right triangles  $\triangle BCD$  and  $\triangle BFE$ , with the other lengths as labeled, find  $x, y$  and  $z$  so that this diagram is possible.

- a)  $x = 5, y = 14, z = 4$     b)  $x = 7, y = 13, z = 3$   
 c)  $x = 5, y = 13, z = 3$     d) none of the previous

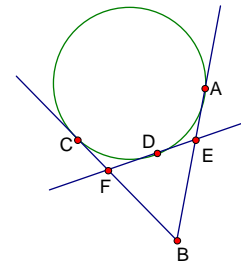


35. The intersection of the 3 angle bisectors of a triangle is called the

- a) orthocenter.    b) centroid.  
 c) circumcenter.    d) none of the previous.

36. Given  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{EF}$  are tangent to the circle with  $AB = BC = 16$ . Find the perimeter of  $\triangle BEF$ .

- a) 30    b) 32  
 c) 34    d) none of the previous

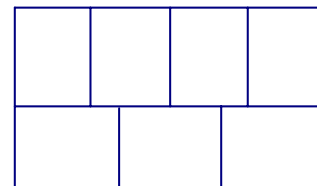


37. If I want to inscribe a circle in a given triangle, the center of the circle would be found by the intersection of the

- a) perpendicular bisectors of the sides.    b) angle bisectors.  
 c) medians.    d) none of the previous.

38. The seven congruent smaller rectangles form a larger rectangle with area  $336 \text{ in}^2$ . Find the perimeter of the larger rectangle.

- a) 70 in    b) 76 in  
 c) 84 in    d) none of the previous



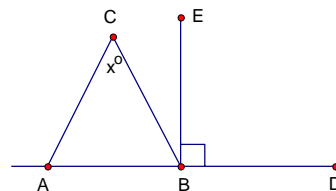
39. Find the surface area of a closed rectangular box with dimensions  $L = 3$  ft,  $W = 5$  ft, and  $H = 4$  ft.
- a)  $47 \text{ ft}^2$     b)  $60 \text{ ft}^2$
- c)  $94 \text{ ft}^2$     d) none of the previous.

40. What is the volume of a sphere with a radius of 4 inches?
- a)  $16 \text{ in}^3$     b)  $\frac{64}{3} \text{ in}^3$
- c)  $64 \text{ in}^3$     d) none of the previous

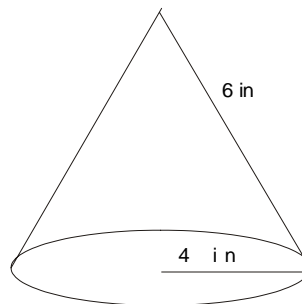
41. What is the total surface area of a cylindrical can that has a base radius of 3 inches and a height of 5 inches?
- a)  $16\pi$  square inches    b)  $31\pi$  square inches
- c)  $34\pi$  square inches    d) none of the previous

42. Given that  $AC = BC$ ,  $\angle EBD = 90^\circ$ , and  $\angle ACB = x^\circ$ , find the measure of  $\angle CBE$  in degrees.

- a)  $x$     b)  $\frac{x}{2}$
- c)  $\frac{x}{3}$     d) none of the previous

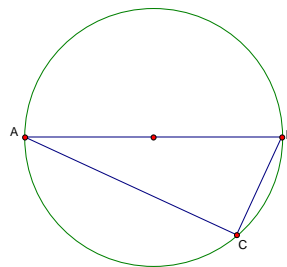


43. Find the volume of a cone with a base radius of 4 inches and a slant height of 6 inches.
- a)  $\frac{32\pi}{3}\sqrt{5}$     b)  $32\pi$
- c)  $32\pi\sqrt{5}$     d) none of the previous

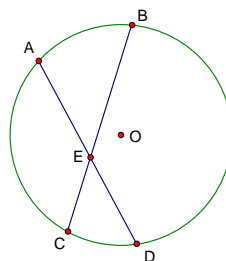


44. Given the circle with diameter  $\overline{AB}$  and  $\widehat{BC} = 76^\circ$ , find the measure of  $\angle B$  in degrees.

- a) 42    b) 52
- c) 62    d) none of the previous

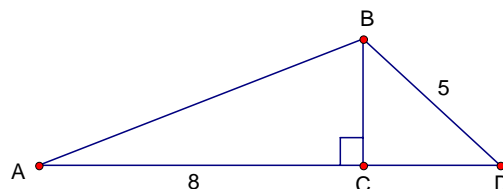


45. Given intersecting chords  $\overline{AD}$  and  $\overline{BC}$  with  $\widehat{AB} = x^\circ$ , and  $\angle AEB = 40^\circ$ , find an expression for the measure of  $\widehat{CD}$  in degrees.



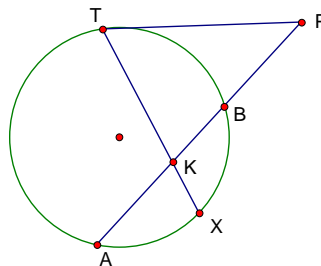
- a)  $40 - x$       b)  $80 - x$   
 c)  $\frac{40 - x}{2}$       d) none of the previous

46. Given that  $\triangle ABC$  has an area of  $12 \text{ cm}^2$ ,  $AC = 8 \text{ cm}$ ,  $BD = 5 \text{ cm}$ , and  $\angle BCD = 90^\circ$ , find the area of  $\triangle BCD$ .



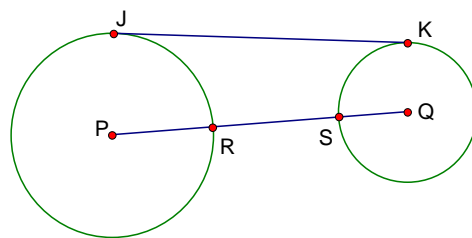
- a)  $6 \text{ cm}^2$       b)  $12 \text{ cm}^2$   
 c)  $18 \text{ cm}^2$       d) none of the previous

47. In the diagram that is not drawn to scale, if  $PT = PK$  and  $\overline{PT}$  is tangent to the circle, find the relationship between  $\widehat{AX}$  and  $\widehat{XB}$ .



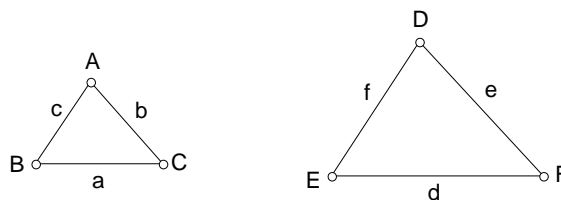
- a)  $\widehat{AX} = \widehat{XB}$       b)  $2\widehat{AX} = \widehat{XB}$   
 c)  $\widehat{AX} = 2\widehat{XB}$       d) none of the previous

48. Given that  $\overline{JK}$  is tangent to the circles  $P$  and  $Q$ ,  $PR = 11$ ,  $RS = 3$ , and  $SQ = 3$ , find  $JK$ .



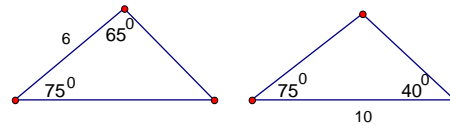
- a) 13      b) 15  
 c) 17      d) none of the previous

49. Given that  $\triangle ABC \sim \triangle DEF$  with ratio  $k < 1$  and the figures are drawn to scale, which one of the following statements is true?



- a)  $\frac{bc}{a} = \frac{k^2ef}{d}$       b)  $\frac{bc}{a} = \frac{ef}{k^2d}$   
 c)  $\frac{bc}{a} = \frac{k^2ef}{d}$       d) none of the previous

50. Which theorem, if possible, can be used to prove the following 2 triangles congruent?



- a) *SSS*      b) *SAS*  
c) *AAS*      d) none of the previous