MERTEN HASSE MATH COMPETITION

Geometry

2017

1. Find the slope of a line perpendicular to $-6x = 5y - 10$.

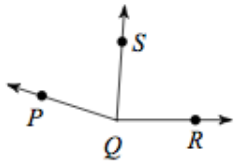
   a) $\frac{6}{5}$

   b) $\frac{5}{6}$

   c) $-\frac{6}{5}$

   d) None of the above

2. Find $m\angle PQS$ if $m\angle PQS = 16 + 5x$, $m\angle PQR = 13x + 6$, and $m\angle SQR = 86°$.



   a) 12°

   b) 76°

   c) 95°

   d) None of the above

3. Find the distance between each pair of points: $(-3\frac{1}{4}, -\frac{6}{5}), (-3\frac{2}{3}, -\frac{3}{4})$

   a) $\frac{\sqrt{1354}}{60}$

   b) $\frac{\sqrt{185914}}{60}$

   c) $\frac{\sqrt{195}}{15}$

   d) None of the above

4. Find the volume of a hexagonal pyramid 8 in tall with a regular base measuring 5 in on each side and an apothem of length 4.3 in. Round your answer to the nearest hundreth, if necessary.

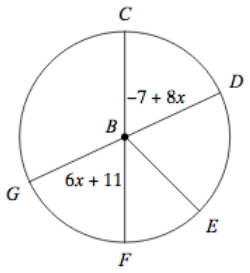
   a) 516 in^3

   b) 28.67 in^3

   c) 172 in^3

   d) None of the above

5. Find the measure of the angle indicated. Assume that the lines which appear to be diameters are actual diameters.
$m\angle CBD$

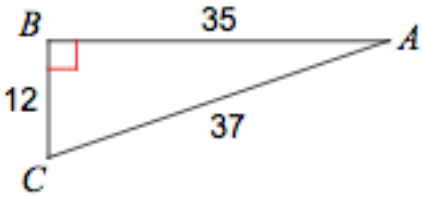


a) 82°
b) 65°
c) 76°
d) None of the above

6. Find the area of a 10-gon with a perimeter of 470 cm.
a) 16996.5 cm^2
b) 17871.2 cm^2
c) 5806.7 cm^2
d) None of the above

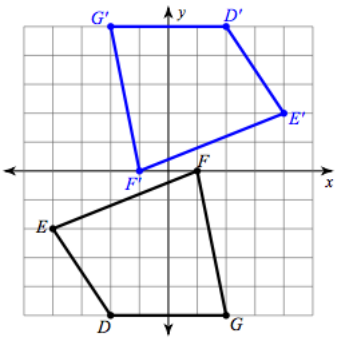
7. Use the information provided to write the equation of the circle.
Ends of a diameter: (-11, -16) and (7, 4)
a) $(x+2)^2+(y+6)^2=181$
b) $(x-2)^2+(y+6)^2=32761$
c) $(x+2)^2+(y+6)^2=36$
d) None of the above

8. Find the value of the trigonometric ratio $cosA$.



a) $\frac{37}{35}$
b) $\frac{35}{37}$
c) $\frac{12}{37}$
d) None of the above

9. Write a rule to describe each transformation.



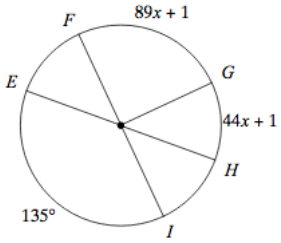
a) reflection across $x = -1$

b) rotation 180° about the origin

c) translation: 3 units right, 5 units up

d) None of the above

10. Find the measure of the arc GH. Assume that the lines which appear to be diameters are actual diameters.



a) 45°

b) 51°

c) 47°

d) None of the above

11. Find the interior angle sum for a regular 13-gon. Round your answer to the nearest tenth if necessary.

a) 2880°

b) 1980°

c) 2520°

d) None of the above

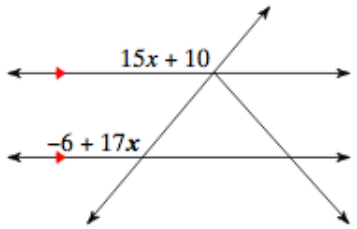
12. The scale factor between two similar figures is given. The surface area and volume of the smaller figure are given. Find the surface area and volume of the larger figure.
scale factor= 3:5
SA=90 in^2
V=729 in^3

a) SA=750 in^2, V=1215 in^3
b) SA=1250 in^2, V=3375 in^3
c) SA=250 in^2, V=3375 in^3
d) None of the above

13. Two sides of a triangle have the following measures. Find the range of possible measures for the third side.
39, 36

a) $3 < x < 74$
b) $3 < x < 75$
c) $4 < x < 75$
d) None of the above

14. Find the measure of the angle indicated in bold.



a) 130°
b) 66°
c) 93°
d) None of the above

15. Find the other endpoint of the line segment with the given endpoint and midpoint.
Endpoint: $(-2\frac{5}{6}, -1\frac{7}{8})$, midpoint: $(3\frac{1}{8}, \frac{1}{2})$

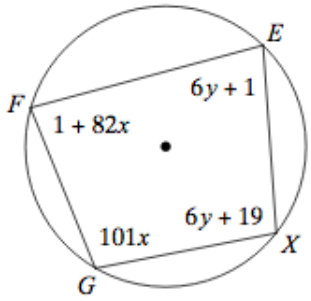
a) $(-2\frac{17}{48}, 1\frac{13}{16})$
b) $(6, -5\frac{1}{2})$
c) $(-2\frac{47}{48}, -1\frac{3}{16})$
d) None of the above

16. Some information about the surface area and volume of two similar solids has been given. Find the missing value.

Solid # 1	Solid # 2
SA=171 yd^2	SA=?
V=243 yd^3	V=72 yd^3

a) $SA = 76$ yd^2
b) $SA = 114$ yd^2
c) $SA = 51$ yd^2
d) None of the above

17. Solve for x and y.



a) $x = 1, y = 4$
b) $x = 1, y = 13$
c) $x = 10, y = 6$
d) None of the above

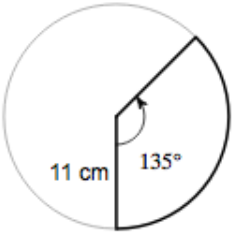
18. Write the standard form of the equation of the line through: (-1,3), perpendicular to y=$\frac{5}{8}x - 2$

a) $5x - 8y = 15$
b) $8x + 5y = -15$
c) $8x + 5y = 7$
d) None of the above

19. Points A, B, C, and D are collinear and positioned in that order. Solve for x if $AD = 89$, $BC = 4$, $AC = 7x + 504$, and $BD = 11x + 795$.

a) -67
b) 37
c) -22
d) None of the above

20. Find the area of the sector.



a) $\frac{363\pi}{8}$ cm^2

b) $\frac{289\pi}{12}$ cm^2

c) $2970cm^2$

d) None of the above

21. The points $(-6, 2)$, $(-2, 6)$, and $(2, 2)$ are vertices of a square. Find the fourth vertex.

a) $(-2, 2)$

b) $(2, -1)$

c) $(4, -2)$

d) None of the above

22. Find D such that $ABCD$ is a parallelogram with $A = (1, -1)$, $B = (0, 2)$, and $C = (6, 5)$.

a) $(7, 3)$

b) $(\frac{15}{5}, 2)$

c) $(7, 2)$

d) None of the above

23. The segment AB lies in the line $x - 4y + 10 = 0$. Its perpendicular bisector is a line passing through $(0, 11)$. Find point B if $A = (-2, 2)$.

a) $(4, 6)$

b) $(6, \frac{9}{5})$

c) $(6, 4)$

d) None of the above

24. In a given triangle a height is constructed from the right angle to the hypotenuse. The triangle then is divided into two smaller triangles. If one of them has three times the area of the other and the hypotenuse of the original triangle has length 1, find the the lengths of its legs.

a) $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$

b) $\frac{5}{4}$ and $\frac{3}{5}$

c) 1 and 2

d) None of the above

25. Find the area of the triangle with vertices $(3, 1)$, $(6, 2)$, and $(0, -4)$.

a) $\sqrt{143}$

b) 12

c) 13

d) None of the above

26. Find the area of the parallelogram $ABCD$ with $A = (1, 1)$, $B = (5, 2)$, and $C = (4, 4)$.

a) $\sqrt{82}$ b) 9

c) 10 d) None of the above

27. Find the barycenter of the triangle with vertices $(-1, -1)$, $(1, 4)$, and $(5, 2)$.

a) $(\frac{5}{3}, \frac{5}{3})$ b) $(2, \frac{11}{6})$

c) $(6, 4), (4, 7)$ d) None of the above

28. Find the shortest distance from the point $(0, 2)$ to the parabola $y = 4 - x^2$.

a) $\sqrt{2}$ b) 1

c) $\frac{7}{5}$ d) None of the above

29. Find the value of c such that the line $2x + 3y = c$ is tangent to the circle $x^2 + y^2 + 6x + 4y = 0$.

a) $\sqrt{3}$ b) $\frac{9}{9}$

c) $\frac{7}{5}$ d) None of the above

30. Find the the equation of the circle of radius $\sqrt{13}$ that is tangent to the circle $x^2 + y^2 - 4x + 2y - 47 = 0$ at the point $(6, 5)$

a) $(x - \frac{15}{2})^2 + (y - 8)^2 = 13$ b) $(x - 7)^2 + (y - 8)^2 = 13$

c) $(x - 8)^2 + (y - 8)^2 = 13$ d) None of the above

31. Find the equations of the lines tangent to the parabola $y^2 + 4x + 2y + 9 = 0$ at the point $(-6, 3)$

a) $-5x - 7x = 9$ and $x + 2y = \frac{5}{6}$ b) $6x + 13y = 0$ and $2x - y = -\frac{31}{2}$

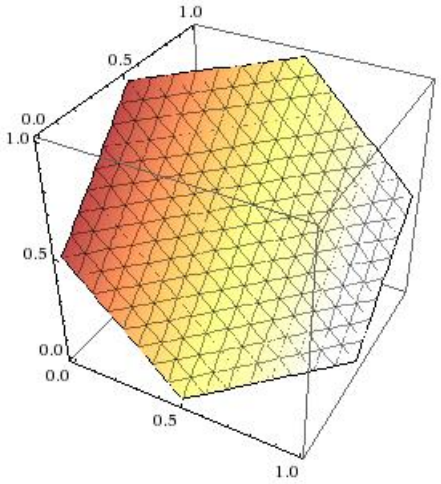
c) $x + 2y = 0$ and $2x - y = -15$ d) None of the above

32. The midpoint of the chord of the ellipse $x^2 + 4y^2 - 6x - 8y - 3 = 0$ is $(5, 2)$. Find the equation of the chord.

a) $x + 2y = 8$ b) $5x + 11y = 9$

c) $x + 2y = 9$ d) None of the above

33. Find the area of the regular hexagon that is the result of a cross-section of a unit cube.



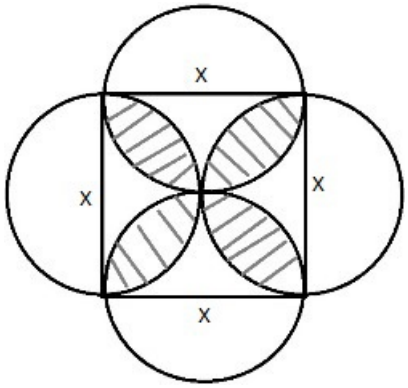
a) $\dfrac{3\sqrt{3}}{2}$

b) $\dfrac{3\sqrt{3}}{4}$

c) $\dfrac{3\sqrt{2}}{2}$

d) None of the above

34. In the picture below $x = 3$. Find the area of the shaded region.



a) $9\pi/2$

b) $9\pi^2/2$

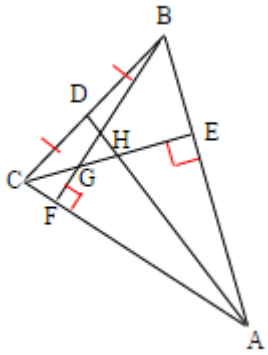
c) $9\pi/2 - 9$

d) None of the above

35. For the frieze pattern below, find all transformations that map the pattern.

a) rotation and translation
b) glide reflection and translation
c) horizontal reflection and translation
d) None of the above

36. What is the orthocenter of the triangle shown below ?



a) Point E
b) Point G
c) Point H
d) None of the above

37. What kind of angle is $\angle 5$ with respect to the picture below ?

a) of depression from Brad to waterfall
b) of depression from waterfall to Brad
c) of elevation from waterfall to Brad
d) None of the above

38. Find the area of a triangle formed by joining the midpoints of the sides of a triangle whose medians have measures 15, 15, and 18 ?

a) 30 b) 36

c) 37 d) None of the above

39. Classify triangle ΔABC with sides a, b, c opposite to angles $\angle A, \angle B, \angle C$ such that $\frac{a+c}{b} = \cot(\frac{B}{2})$.

a) right b) equilateral

c) obtuse d) None of the above

40. Point P is located in the interior of square ABCD such that PA=3, PB=7, and PD=5. What is the area of the square ABCD ?

a) 60 b) 58

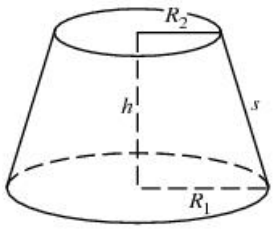
c) 48 d) None of the above

41. To what familiar result does Ptolemy's theorem lead when the cyclic quadrilateral is a rectangle ?

a) Menelaus' theorem b) SSS postulate

c) law of cosines d) None of the above

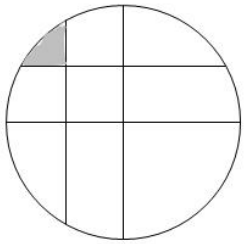
42. The conical frustum below is dipped in paint and rolled (without slipping) such that it leaves a closed path of paint. Find the area of the path in terms of R_1, R_2, and s.



a) $\pi s^2 \cdot \dfrac{R_1 + R_2}{R_1 - R_2}$ b) $\pi s^2 \cdot \dfrac{R_1 R_2}{R_1 + R_2}$

c) $\pi s \cdot (R_1^2 - R_2^2)$ d) None of the above

43. Consider a circle whose radius is 1 with two perpendicular diameters. Along either diameter, a segment has been drawn perpendicular to one of its radii such that it bisects that radius, as seen in the image. Find the area of the shaded region.



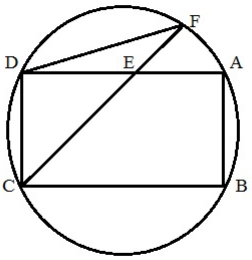
a) $\frac{\pi+2-3\sqrt{2}}{12}$

b) $\frac{\pi+3-3\sqrt{3}}{12}$

c) $\frac{\pi+3-\sqrt{6}}{36}$

d) None of the above

44. E is a point on side AD of rectangle ABCD, so that DE=6, while DA=8, and DC=6. If CE extended meets the circumcircle of the rectangle at F, find the measure of chord DF.



a) $4\sqrt{2}$

b) 5

c) $5\sqrt{2}$

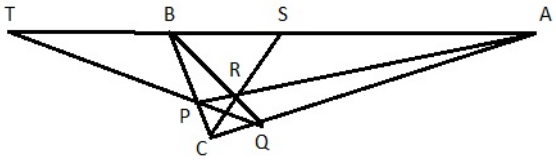
d) None of the above

45. A regular pentagon ABCDE is inscribed in a circle, and point P is on $\overset{\frown}{BC}$. Which of the following is true ?

a) $PA + PD < PB + PC + PE$ b) $PA + PD = PB + PC + PE$

c) $PA + PD > PB + PC + PE$ d) None of the above

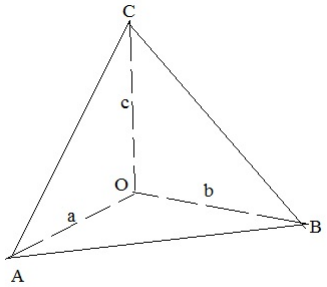
46. In ΔABC angle $\angle C$ is right, P and Q are on $\overline{BC}$ and $\overline{AC}$, respectively, such that CP=CQ=2. Through the point of intersection, R, of $\overline{AP}$ and$\overline{BQ}$, a line is drawn also passing through C and meeting $\overline{AB}$ at S. $\overline{PQ}$ extended meets line $\overleftrightarrow{AB}$ at T. If hypotenuse AB=10 and AC=8, find TS.



a) 33/2 b) 20

c) 24 d) None of the above

47. Lines $\overleftrightarrow{OA}$, $\overleftrightarrow{OB}$, and $\overleftrightarrow{OC}$ are mutually perpendicular. What is the measure of the angle between planes (ABC) and (OAB) if $OA = a$, $OB = b$, and $OC = c$?



a) $\tan^{-1}\left(\frac{c\sqrt{a^2+b^2}}{ab}\right)$ b) $\tan^{-1}\left(\frac{\sqrt{a^2+b^2}}{abc}\right)$

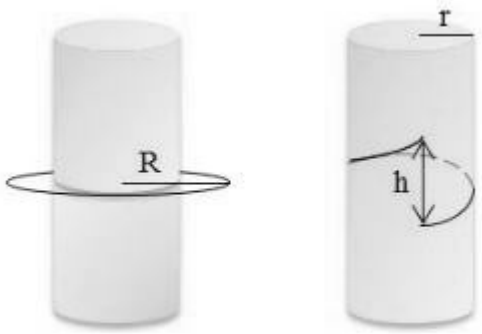
c) $\tan^{-1}\left(\frac{abc}{\sqrt{a^2+b^2}}\right)$ d) None of the above

48. Let n be a positive integer, and consider the cube C made of n^3 unit cubes. Assume C has no top surface. Now let N be the number of unit cubes that are touching either a side or the bottom of C. What is the value of N in terms of n?

a) $4n^2 + 2n + 1$ b) $3n^2 - 2n + 4$

c) $5n^2 - 8n + 4$ d) None of above

49. Suppose we have an annular wire of radius R and a cylindrical rod of radius $r < R$. The wire is cut and tightly wrapped around the rod once so that the segment of length h in the figure below is vertical. Assuming that the wire does not stretch find the length h.



a) $2\pi\sqrt{R^2 - r^2}$

b) $2\pi(R - r)$

c) $\pi\sqrt{R^2 - r^2}$

d) None of the above

50. A circular coin of radius r is thrown on a floor made of square tiles of size $L = 10r$. What is the probability that the coin would land entirely within a single tile, without touching one of the cracks ?

a) 86%

b) 64%

c) 100%

d) None of the above

Answer Key

1. b) $\frac{5}{6}$
2. b) $76°$
3. a) $\frac{\sqrt{1354}}{60}$
4. c) 172 in^3
5. b) $65°$
6. a) 16996.5 cm^2
7. a) $(x+2)^2+(y+6)^2=181$
8. b) $\frac{35}{37}$
9. b) rotation $180°$ about the origin
10. a) $45°$
11. b) $1980°$
12. c) SA=250 in^2, V=3375 in^3
13. b) $3<x<75$
14. a) $130°$
15. d) None of the above
16. a) $SA = 76$ yd^2
17. b) $x=1, y=13$
18. c) $8x+5y=7$
19. a) -67
20. a) $\frac{363\pi}{8}$ cm^2
21. a) $(-2,2)$
22. c) $(7,2)$
23. c) $(6,4)$
24. a) $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$
25. b) 12
26. b) 9
27. a) $(\frac{5}{3},\frac{5}{3})$
28. a) $\sqrt{2}$
29. d) None of the above
30. c) $(x-8)^2+(y-8)^2=13$
31. c) $x+2y=0$ and $2x-y=-15$
32. c) $x+2y=9$
33. b) $\frac{3\sqrt{3}}{4}$
34. c) $9\pi/2-9$
35. a) rotation and translation
36. b) Point G
37. d) None of the above
38. b) 36
39. a) right
40. b) 58
41. c) law of cosines

42. a) $\pi s^2 \cdot \dfrac{R_1 + R_2}{R_1 - R_2}$

43. b) $\frac{\pi+3-3\sqrt{3}}{12}$

44. c) $5\sqrt{2}$

45. b) $PA + PD = PB + PC + PE$

46. c) 24

47. a) $\tan^{-1}\left(\frac{c\sqrt{a^2+b^2}}{ab}\right)$

48. c) $5n^2 - 8n + 4$

49. a) $2\pi\sqrt{R^2 - r^2}$

50. b) 64%