

Merten Hasse Math Competition
Division Precalculus
Senior Math

2017

1. Perform the indicated operations and simplify the expression: $\frac{3x + 3y}{x^{-2} - y^{-2}}$.
- a) $\frac{3}{y - x}$ b) $\frac{3x^2y^2}{x + y}$
c) $\frac{-3x^2y^2}{x - y}$ d) none of the above
2. Factor and simplify completely: $(5x - 2)^{1/2} - \frac{1}{3}(5x - 2)^{3/2}$
- a) $-\frac{1}{3}(5x - 2)^{1/2}(5x - 1)$ b) $-\frac{5}{3}(5x - 2)^{1/2}(x - 1)$
c) $-\frac{5}{3}(5x - 2)^{1/2}(x + 1)$ d) none of the above
3. Solve for y : $7x^{-1/2} + 2x^{3/2}y = 3x^{-1/2}(y + 4)$.
- a) $y = \frac{-3}{2x^2 - 3}$ b) $y = \frac{12x^{1/2} - 7}{2x^{3/2} - 3}$
c) $y = \frac{5}{2x^2 - 3}$ d) none of the above
4. Find $\frac{F(x + h) - F(x)}{h}$, $h \neq 0$, for $F(x) = \frac{3}{(x - 1)^2}$.
- a) $3(2 - 2x - h)$ b) $\frac{3(2 - 2x - h)}{(x + h - 1)^2(x - 1)^2}$
c) $\frac{-3(h^2 + 2)}{h(x + h - 1)^2(x - 1)^2}$ d) none of the above
5. Which of the following inequalities is equivalent to $|4x - 3| < 0.02$?
- a) $4x - 3 < 0.02$ b) $4x - 3 < 0.02$ or $4x - 3 > -0.02$
c) $-0.02 < 4x - 3 < 0.02$ d) none of the above
6. Solve for z in: $8(x^2 + y^2)(x + yz) = 50(x - yz)$
- a) $z = \frac{25x - 4x(x^2 + y^2)}{4y(x^2 + y^2) + 25y}$ b) $z = \frac{25x}{4(x^2 + y^2)(x + y) + 25y}$
c) $z = \frac{25x - 4x(x^2 + y^2)}{2y}$ d) none of the above
7. Solve: $x^{2/3} - 6x^{1/3} = 7$
- a) 21, -3 b) 7, -1
c) 343, -1 d) none of the above
8. Solve the inequality: $\frac{1}{3} \leq \log x < 1$
- a) $-30 \leq x < 10$ b) $\sqrt{10} + \frac{1}{3} \leq x < 10$
c) $\sqrt[3]{10} \leq x < 10^3$ d) none of the above

9. Let $z_1 = 4(\cos(\pi/6) + i \sin(\pi/6))$ and $z_2 = 5(\cos(\pi/3) + i \sin(\pi/3))$. Find $z_1 z_2$.

- a) $20 - 20i$ b) $20i$
c) $20 + i$ d) none of the above

10. If $f(x) = x^2 - 1$ and $g(x) = \sqrt{x} + 1$, find $(f \circ g)(x)$.

- a) x b) \sqrt{x}
c) $x + 2\sqrt{x}$ d) none of the above

11. Solve the equation: $625^{x-4} = 25^{x+5}$

- a) 14 b) 13
c) -13 d) none of the above

12. Find the solution z of the system:

$$\begin{cases} x + 2y + 12z = 67 \\ x + 4y + 22z = 121 \\ -x - z = -8 \end{cases}$$

- a) 2 b) 3
c) 5 d) none of the above

13. Solve the inequality $(x + 2)^2 \geq x^2 + 2^2$

- a) $[0, \infty)$ b) $(-\infty, \infty)$
c) $(-\infty, 0)$ d) none of the above

14. Find the inverse function of $f(x) = (x - 10)^2$, where $x \leq 10$

- a) $f^{-1}(x) = \sqrt{x} + 10$ b) $f^{-1}(x) = -\sqrt{x} + 10$
c) $f^{-1}(x) = \sqrt{x + 10}$ d) none of the above

15. Describe how the graph of $y = -f(2x) - 4$ can be obtained from the graph of f .
- Shrink horizontally by a factor of $1/2$, reflect about y-axis, shift 4 units down;
 - Shrink horizontally by a factor of $1/2$, reflect about x-axis, shift 4 units right;
 - Shrink horizontally by a factor of 4, reflect about x-axis, shift 2 units up;
 - none of the above
16. Given that $f(x) = \cos x$, find the average rate of change of f from $x_1 = \pi/6$ to $x_2 = 2\pi/3$.
- $\frac{1 - \sqrt{3}}{\pi}$
 - $\frac{1 + \sqrt{3}}{4}$
 - $\frac{-1 - \sqrt{3}}{\pi}$
 - none of the above
17. The range of the piecewise function defined by $f(x) = \begin{cases} (x - 2)^2 & \text{if } x < 3 \\ 2x - 5 & \text{if } x > 3 \end{cases}$ is:
- $(-\infty, \infty)$
 - $[0, 1) \cup (1, \infty)$
 - $(1, \infty)$
 - none of the above
18. The remainder by division of the polynomial $f(x) = x^{2017} - 2017x$ by $g(x) = x + 1$ is:
- 2017
 - 2016
 - 2016
 - none of the above
19. Which of the following are asymptote equations for $f(x) = \frac{12(7x - 4)(x + 1)}{9x^2 - 14x - 23}$
- $y = \frac{28}{3}, x = \frac{23}{9}$
 - $y = \frac{4}{3}, x = \frac{23}{9}, x = -1$
 - $y = 0, x = \frac{23}{9}$
 - none of the above
20. What is the y-intercept of the line which passes through the ordered pair point $(-1, 3)$ and is perpendicular to the line whose equation is $y = \frac{-7}{26}x + \frac{4}{5}$?
- $\frac{-5}{7}$
 - $\frac{47}{7}$
 - $\frac{71}{26}$
 - none of the above

21. The function h is given by $h(x) = \log_2(x^2 + 2)$. For what positive value of x does $h(x) = 3$?
- a) 1 b) $\sqrt{6}$
- c) $\sqrt{7}$ d) none of the above
22. Consider the function $f(x) = \tan(\ln(\sin x))$. Denote by \mathcal{D} its domain. Then:
- a) \mathcal{D} contains intervals of arbitrarily small length b) \mathcal{D} is bounded
- c) \mathcal{D} contains intervals of arbitrarily large length d) none of the above
23. Let $f(x) = \frac{x-1}{x}$. Consider the composition of $f(x)$ with itself 2017 times. The result will be:
- a) $\frac{x^{2017}-1}{x^{2017}}$ b) $\frac{x-1}{x}$
- c) x d) none of the above
24. Find the center of the circle whose equation is $x^2 + y^2 + 3x - 8y - 8 = 0$.
- a) $\left(\frac{3}{2}, 4\right)$ b) $(-3, 8)$
- c) $\left(-\frac{3}{2}, 4\right)$ d) none of the above
25. Let $a = 11.3$ cm, $b = 10.3$ cm, and $c = 16.8$ cm. Find the area of the triangle with the given parts, rounded to the closest integer.
- a) 60 cm^2 b) 63 cm^2
- c) 57 cm^2 d) none of the above
26. A rectangular piece of land whose length is twice its width has a diagonal distance of 90 yards. Which one of the following represents the distance a person would save by walking diagonally across the land instead of walking its length and its width?
- a) $54\sqrt{5} - 90$ b) $90\sqrt{3} - 90$
- c) $90 - 18\sqrt{5}$ d) none of the above

27. A steel storage tank for propane gas is in the shape of a right circular cylinder of altitude 10 feet with a hemisphere attached to each end. Which one of the following expresses the surface area, S , of the tank as a function of its radius, r .

a) $S(r) = 4\pi r^2 + 20\pi r$ b) $S(r) = \frac{4}{3}\pi r^2 + 20\pi r$

c) $S(r) = 60\pi r^2 + 20\pi r$ d) none of the above

28. A piece of wire 10 meters long is cut into two pieces. Let x represent the length of the cut. The piece of length x is bent into a square. The remaining piece is bent into an equilateral triangle. Express the total area, A , enclosed by the square and the equilateral triangle as a function of x .

a) $A(x) = \frac{x^2}{4} + \frac{(10-x)^2}{9}$ b) $A(x) = \frac{x^2}{16} + \frac{\sqrt{3}(10-x)^2}{18}$

c) $A(x) = \frac{x^2}{16} + \frac{\sqrt{3}(10-x)^2}{36}$ d) none of the above

29. Determine the total number of solutions of the equation $2\sin^4 x - 3\sin^3 x - 3\sin x - 2 = 0$ in the interval $[0, 2\pi]$.

a) 2 solutions b) 4 solutions

c) 3 solutions d) none of the above

30. A contractor is to build a warehouse whose rectangular floor will have an area of 6000 square feet. The warehouse will be separated into two equal rectangular rooms by an interior wall. The cost of the exterior walls is \$225 per linear foot and the cost of the interior wall is \$100 per linear foot. Denote by x the length of the exterior wall separated in half by the interior wall. Express the contractor's cost for building the walls, C , as a function of x .

a) $C(x) = 450x + \frac{6000}{x}$ b) $C(x) = 450x + \frac{3300000}{x}$

c) $C(x) = 450x + \frac{2700000}{x}$ d) none of the above

31. Determine the number of solutions of the equation:

$$\underbrace{\cos(\cos(\dots(\cos x)\dots))}_{\text{composition 2017 times}} = 0.$$

a) no solutions

b) 1 solution

c) there is an infinite number of solutions d) none of the above

32. The measure of a certain angle is 25° . What is the corresponding radian measure of the angle?
- a) $\frac{5\pi}{9}$ b) $\frac{5\pi}{18}$
c) $\frac{5\pi}{36}$ d) none of the above
33. If $\frac{\pi}{2} < \theta < \pi$ and $10 \sin \theta = z$, what is the $\tan \theta$ in terms of z ?
- a) $-\frac{z}{\sqrt{100 - z^2}}$ b) $\frac{10}{\sqrt{z^2 - 100}}$
c) $\frac{z}{\sqrt{100 - z^2}}$ d) none of the above
34. Consider $\csc(\theta) > 0$ and $\sec(\theta) > 0$. Name the quadrant the angle θ lies.
- a) Quadrant II b) Quadrant III
c) Quadrant IV d) none of the above
35. Simplify $\sec^6(x) - 3 \sec^4(x) \tan^2(x) + 3 \sec^2(x) \tan^4(x) - \tan^6(x)$.
- a) 1 b) $\sec^2(x)[1 + \tan^2(x)]$
c) $\sec^3(x) - \tan^3(x)$ d) none of the above
36. An airplane is sighted at the same time by two ground observers who are 2 miles apart and in line with the airplane. They report the angles of elevation as 10° and 22° . How high is the airplane in the air?
- a) 0.63 miles b) 0.35 miles
c) 0.75 miles d) none of the above
37. For $\tan \theta = \frac{21}{20}$, where $180^\circ < \theta < 270^\circ$, then $\cos(\theta/2)$ is:
- a) $\frac{-3\sqrt{58}}{58}$ b) $\frac{-10}{29}$
c) $\frac{-7\sqrt{58}}{58}$ d) none of the above
38. Find the exact value for $\cos^{-1}\left(\cos \frac{5\pi}{3}\right)$
- a) $\frac{5\pi}{3}$ b) $\frac{1}{2}$
c) $\frac{\pi}{6}$ d) none of the above

39. 21. What are the amplitude and the period of the function $f(x) = 3 + 15 \cos(6x) \cos(7) - 15 \sin(6x) \sin(7)$?
- a) Amplitude: 3 Period $\frac{\pi}{7}$ b) Amplitude: 15 Period $\frac{\pi}{3}$
- c) Amplitude: 15 Period $\frac{\pi}{6}$ d) none of the above
40. A pilot sets out from an airport and heads in the direction N 30 W, flying at a constant speed of 305 mph. Forty-five minutes later the pilot makes a course and speed correction and now heads in the direction N 50 W and reduces her speed to 175 mph. Half an hour later, engine trouble forces her to make an emergency landing. Find the distance between the airport and her final landing point.
- a) 408.8 miles b) 312.41 miles
- c) 216.41 miles d) none of the above
41. Rewrite the expression $\sin(\sin^{-1} x - \cos^{-1} x)$ as an algebraic expression in x .
- a) x^2 b) $2x$
- c) $2\sqrt{1-x^2}$ d) none of the above
42. The SUM of the solutions of the equation: $2 - 2e^t + e^{2t} = e^{-t} + 6e^{-2t}$ is:
- a) 5π b) $\frac{7\pi}{2}$
- c) 3π d) none of the above
43. The annual yield per lemon tree is fairly constant at 400 pounds per tree when the number of trees per acre is 125 or fewer. For each additional tree over 125, the annual yield per tree for all trees on the acre decreases by 5 pounds due to overcrowding. Express the total yield for an acre, T , in pounds, as a function of the number of lemon trees per acre, x , where x is greater than 125.
- a) $T(x) = 275 - 5x^2$ b) $T(x) = 1025x - 5x^2$
- c) $T(x) = 5x^2 - 125x$ d) none of the above

44. Find the solutions of the equation: $\log \frac{x+1}{x} + \log x + 2x + 1 + \dots +$

$$\log \frac{x+2016}{x+2015} = 1.$$

a) 1 and 473 b) 224
c) 2017 d) none of the above

45. On a certain route, an airline carries 8000 passengers per month, each paying \$70. A market survey indicates that for each \$1 increase in the ticket price, the airline will lose 50 passengers. What is the maximum monthly revenue?

- a) \$661,250 b) \$920,000
c) \$568,000 d) none of the above

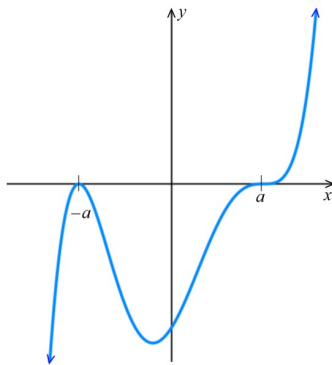
46. Write a recursion formula for the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

- a) $a_{n-1} = \frac{1}{2}a_n$ b) $a_n = \frac{1}{2}a_{n-1}$
c) $a_n = (a_{n-1}) - \frac{1}{2}$ d) none of the above

47. Find the sum: $\sum_{k=4}^{30} (100 - 5k)$

- a) 405 b) 435
c) 455 d) none of the above

48. Which of the following functions is represented by the graph shown below?



- a) $f(x) = (x-a)^2(x+a)^3$ b) $f(x) = (x-a)^3(x+a)^2$
c) $f(x) = (x-a)^2(x+a)^2$ d) none of the above

49. The SUM of the solutions of the equation: $2 - 2e^t + e^{2t} = e^{-t} + 6e^{-2t}$ is:

a) $\ln 8$ b) $\ln 12$

c) $\ln 2$ d) none of the above

50. FUN FACTS: Consider the number 2017. Multiply this number by π and round to the nearest integer. Then multiply 2017 by e and round to the nearest integer. You obtain in this way three numbers that have a common property:

a) they are all divisible by 3 b) they are all divisible by 7

c) they are all prime d) none of the above

Solutions' key

- | | |
|----------------|----------------|
| 1. <i>c</i>) | 26. <i>a</i>) |
| 2. <i>b</i>) | 27. <i>a</i>) |
| 3. <i>c</i>) | 28. <i>c</i>) |
| 4. <i>b</i>) | 29. <i>a</i>) |
| 5. <i>c</i>) | 30. <i>b</i>) |
| 6. <i>a</i>) | 31. <i>a</i>) |
| 7. <i>c</i>) | 32. <i>c</i>) |
| 8. <i>d</i>) | 33. <i>a</i>) |
| 9. <i>b</i>) | 34. <i>d</i>) |
| 10. <i>c</i>) | 35. <i>a</i>) |
| 11. <i>b</i>) | 36. <i>a</i>) |
| 12. <i>c</i>) | 37. <i>a</i>) |
| 13. <i>d</i>) | 38. <i>d</i>) |
| 14. <i>b</i>) | 39. <i>b</i>) |
| 15. <i>d</i>) | 40. <i>b</i>) |
| 16. <i>c</i>) | 41. <i>d</i>) |
| 17. <i>d</i>) | 42. <i>c</i>) |
| 18. <i>c</i>) | 43. <i>b</i>) |
| 19. <i>a</i>) | 44. <i>b</i>) |
| 20. <i>b</i>) | 45. <i>a</i>) |
| 21. <i>b</i>) | 46. <i>b</i>) |
| 22. <i>a</i>) | 47. <i>a</i>) |
| 23. <i>b</i>) | 48. <i>b</i>) |
| 24. <i>c</i>) | 49. <i>c</i>) |
| 25. <i>c</i>) | 50. <i>c</i>) |